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A NUMERICAL APPROACH FOR FAULT DETECTION IN BEAM-LIKE STRUCTURES USING MODAL STRAIN ENERGY

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ABSTRACT

Vibration-based damage detection is based on the fact that measurements of changes in the vibration properties of a structure can be used to determine the occurrence of structural damage. Therefore, there is a strong need for the development and implementation of fault detection and optimum sensor location methods that address the issue of limited instrumentation and its effect in predicting damage degradation. In this paper, the results of a numerical implementation of a previously developed sensor distribution method are presented. A finite element model of a railway box-girder bridge is assembled. Then, the damage index method is numerically implemented in order to identify damage from the resulting sensor measurements. Both the optimum sensor placement and the damage detection methods are based on the concept of modal strain energy. Finally, the numerical results show that the proposed approach is suitable for continuous damage monitoring implementations and can be used as a useful tool for damage assessment in beam-like structures.

KEY WORDS: structural health monitoring; fault detection; optimum sensor location; modal strain energy; sensitivity matrix; curvature of the vibration mode shape.

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UN ENFOQUE NUMÉRICO PARA LA DETECCIÓN DE FALLAS EN ESTRUCTURAS TIPO VIGA CON USO DE LA ENERGÍA MODAL DE DEFORMACIÓN

RESUMEN

La detección de daño estructural usando vibraciones se fundamenta en el hecho de que pueden usarse mediciones de cambios en las propiedades de vibración de una estructura para determinar la ocurrencia de daño estructural, por lo tanto, existe una fuerte necesidad de desarrollar e implementar métodos de detección de daño y óptima localización de sensores que tengan en cuenta el uso de instrumentación limitada y su efecto en la predicción de deterioro estructural. En este artículo se presentan los resultados de la implementación numérica de un método de óptima localización de sensores previamente desarrollado. Se ensambla un modelo en elementos finitos con base en un modelo de un puente ferroviario de tipo viga-cajón. Luego, el método de índice de daño se emplea para la identificación de daño estructural, con base en las mediciones obtenidas de los sensores. Los métodos de óptima localización de sensores y de detección de daño presentados en este artículo se basan en el uso de la energía modal de deformación. Los resultados numéricos muestran que la energía modal de deformación es apropiada para aplicaciones de seguimiento continuo de daño en estructuras y que puede usarse como una herramienta útil para inspecciones de integridad estructural en estructuras tipo viga.

PALABRAS CLAVE: monitoreo de salud estructural; detección de daño; localización de sensor óptima; energía modal de deformación; matriz de sensibilidad; curvatura de los modos de vibración.

UM ENFOQUE NUMÉRICO PARA A DETECÇÃO DE FALHAS EM ESTRUTURAS TIPO VIGA COM USO DA ENERGIA MODAL DE DEFORMAÇÃO

RESUMO

A deteção de dano estrutural usando vibrações se fundamenta no fato de que podem usar-se medições de mudanças nas propriedades de vibração de uma estrutura para determinar a ocorrência de dano estrutural, portanto, existe uma forte necessidade de desenvolver e aplicar métodos de deteção de dano e ótima localização de sensores que levem em conta o uso de instrumentação limitada e seu efeito no prognóstico de deterioração estrutural. Neste artigo se apresentam os resultados da aplicação numérica de um método de ótima localização de sensores previamente desenvolvido. Encaixa-se um modelo em elementos finitos com base em um modelo de uma ponte ferroviária de tipo viga-gaveta. Depois, o método de índice de dano se emprega para a identificação de dano estrutural com base nas medições obtidas dos sensores. Os métodos de ótima localização de sensores e de detecção de dano apresentados neste artigo estão baseados no uso da energia modal de deformação. Os resultados numéricos mostram que a energia modal de deformação é apropriada para aplicações de seguimento contínuo de dano em estruturas e que pode usar-se como uma ferramenta útil para inspeções de integridade estrutural em estruturas tipo viga.

PALAVRAS CÓDIGO: monitoração de saúde estrutural; deteção de dano; localização de sensor ótima; energia modal de deformação; matriz de sensibilidade; curvatura dos modos de vibração.

1. INTRODUCTION

Continuous damage monitoring of civil engineering structures is based on the premise that damage will cause changes in the measured modal parameters such as eigenfrequencies, mode shapes, and damping ratios. Therefore, vibration-based damage detection methods utilize measured modal parameters to assess the health condition of a structure (Kumar, Shenoi and Cox, 2009). Many of such methods have been developed for civil engineering structures due to their potential to enhance or even eventually replace current visual inspection techniques resulting in significant economic savings. Nevertheless, there are several challenges yet remaining to optimally design a robust continuous damage monitoring system. One of these challenges is the effective placement of sensors to achieve the objectives of the structural health monitoring system (Li, 2010). Optimum sensor placement deals with both the number of sensors needed for economic implementation and the best location of those sensors; in addition, the resulting sensor configuration should consider the modes of interest, the characteristics of the excitation, including the source, type and frequency range and the likelihood of damage in various regions of the structure, if this information is available. Furthermore, the effectiveness of a placement methodology will be strongly related to the requirements and needs of the damage detection technique that is to be employed in the system (Riveros *et al.*, 2008).

On the other hand, although continuous damage monitoring of civil engineering structures has generated a lot of research over the past few years, there is still the debate on how the problem of test/analysis mismatch –in terms of degrees of freedom– can be tackled (Yang, 2009). It is known from sensitivity studies using finite element models and from in-situ tests of artificially damaged bridges that the decrease of frequencies is relatively low. Even though the local stiffness drop at a local damage site may be high, the global stiffness results in relative small frequency changes, which can only be detected with very precise sensing systems and identification procedures. A well-known test program corresponds to the damage tests on the I-40 plate girder bridge located in New Mexico (USA). As reported by Farrar and Jauregui (1996), five modal-based methods for damage detection were compared using data recorded from this bridge where damage was intentionally inflicted. Damage detection results generally showed that the damage index method performed the best. However, in two out of five levels of inflicted damage, the variation of temperature induced an increase of the fundamental frequency of the bridge leading to an undetectable reduction of stiffness. However, Limongelli (2010) used the previously mentioned I-40 plate girder bridge to study the sensitivity to changing temperature of the interpolation damage detection method (IDDM) showing that the IDDM provides reliable results, almost not affected by temperature changes, in case of signals with a low level of noise; for noisy signals, Limongelli (2010) proposed a threshold value in order to judge if the structure had suffered damage.

Alvandi and Cremona (2006) reported good agreement between the predicted damage location and the actual ones for all experimental and simulated damaged cases. They used a simple supported beam under various levels of noise in the applied excitation force and concluded that, regarding noisy signals, the strain energy-based method performed the best. More recently, Li (2010) presented a damage identification study using various strain-related indicators, such as strain (curvature) mode shape, strain energy, strain frequency response function, etc., and concluded that the strain energy-based index demonstrates an excellent anti-noise performance, and therefore improvement of damage identification results can be achieved by using embedding machine learning techniques.

This paper presents a modal strain energy-based approach for identification of structural damage in beam-like structures. A finite element (FE) model of a box-girder bridge is assembled based on data provided by Yang and Wu (2002). Then, an optimum sensor location algorithm is presented and numerically implemented in the FE model. This algorithm is based on the use of the sensitivity matrix, which is derived from mode shapes derivatives. Finally, a modal strain energy-based method is utilized to locate damaged sites.

2. PROBLEM FORMULATION

Optimum sensor placement will be dependent on several factors, including: a) the damage detection algorithm employed; b) the location of damage and relative likelihood of damage in the various locations of the structure; c) excitation source; d) objectives of the continuous damage monitoring system; and e) the selected mode shapes. Herein, we employ the damage index (DI) method for determination of the existence of damage (Stubbs and Kim, 1994), and a previously proposed technique is additionally considered to place sensors, known as the damage measurability (DM) method. The optimum sensor placement method is described in the following section, and this discussion is followed by a description of the DI method.

3. DAMAGE MEASURABILITY METHOD

Several researchers have made significant contributions to the development of methods for optimum sensor placement. One of these is the effective independence method developed by Kammer (1991), which is based on the maximization of the determinant of the Fisher information matrix (FIM). Hemez and Farhat (1994) extended the effective independence method by placing sensors based on the strain energy contributions of the structure; they proposed the use of the FIM as a distribution of strain energy, which yields an array of sensors optimally located to detect structural damage. Udwadia (1994) proposed the optimum sensor location algorithm, which minimizes the covariance error between the structural parameters that are to be identified and their estimate from the limited measurements; this method is rigorously founded on the FIM and is applicable to both linear and nonlinear systems; additionally, it was also proved that the kinetic energy criterion for optimum sensor placement is inappropriate to develop such type of methods. Heredia-Zavoni and Esteva (1998) stated that the optimum sensor configuration should be chosen as the one that minimizes the expected Bayesian loss function involving the trace of the inverse of the FIM. Cobb and Liebst (1996) developed the eigenvector sensitivity method, which employs the eigenvalue and eigenvector sensitivities for the determination of the optimal sensor locations. Shi, Law and Zhang (2000) presented a method in which the sensor locations are selected according to their ability to localize structural damage based on the eigenvector sensitivity method. Xia and Hao (2000) proposed measurement selection in terms of two factors, namely, the sensitivity of a residual vector to the structural damage using the approach proposed by Shi, Law and Zhang (2000) and the sensitivity of the damage to the measurement noise. Riveros *et al.* (2004) numerically implemented the method developed by Xia and Hao (2000) in cable-stayed bridges.

The DM method was developed by Xia and Hao (2000), and is defined using two factors. The first factor is related to the fact that sensor locations must be optimized for the purpose of localizing structural damage sites and the second factor is the sensitivity of damage to measurement noise. The mathematical derivation of the method is based on the sensitivity-based element-by-element (SB-EBE) model updating method proposed by Hemez and Farhat (1994), which represents the relationship between the measured eigenfrequencies and eigenvectors and the structural stiffness parameters using a truncated Taylor series expansion as shown in Eq. (1).

$$\begin{Bmatrix} \tilde{\lambda}_i \\ \tilde{\phi}_i \end{Bmatrix} = \begin{Bmatrix} \lambda_i \\ \phi_i \end{Bmatrix} + [\mathbf{S}_i](\{\tilde{\alpha}\} - \{\alpha\}) \tag{1}$$

where λ_i, ϕ_i and $\tilde{\lambda}_i$, $\tilde{\phi}_i$ are the ith eigenfrequency and eigenvector of the undamaged and damaged structure, respectively, $\{\alpha\}$ and $\{\tilde{\alpha}\}$ are structural elemental stiffness parameters (ESP) of the initial and updated FE model after the occurrence of damage with *ne* elements, and $\mathbf{S_i}$ is the corresponding sensitivity matrix defined in Eq. (2).

$$[\mathbf{S}_i] = \begin{bmatrix} \dfrac{\partial \lambda_i}{\partial \alpha} \\ \dfrac{\partial \phi_i}{\partial \alpha} \end{bmatrix} \tag{2}$$

To estimate the elemental stiffness parameter change according to Eq. (1), the covariance matrix of the estimation errors must be minimized. Udwadia and Garba (1985) demonstrated that maximizing the determinant of the FIM as a distribution of strain energy $\mathbf{B_i}$ given in Eq. (3) for the ith mode

$$[\mathbf{B_i}] = [\mathbf{S_i}]^T [\mathbf{S_i}] \tag{3}$$

leads to the minimization of the covariance matrix and, thus, the best estimate of $\{\tilde{\alpha}\} - \{\alpha\}$. The FIM as a distribution of strain energy $\mathbf{B}$ is defined by the contribution of the selected modes. Kammer (1991) showed that the diagonal terms of

$$\mathbf{E}_\mathrm{i} = ([\mathbf{S}_\mathrm{i}]\left[[\mathbf{S}_\mathrm{i}]^\mathrm{T}[\mathbf{S}_\mathrm{i}]\right]^{-1}[\mathbf{S}_\mathrm{i}]^\mathrm{T}) \tag{4}$$

can be used to rank the importance of a particular DOF for the ith mode. Thus, if a particular DOF has a small contribution to the diagonal terms of $\mathbf{E_i}$, this sensor position can be eliminated from the selected sensor locations, then the remaining sensor locations maximize the contribution to the FIM as a distribution of the strain energy $\mathbf{B_i}$ providing the most information for damage detection. The contribution of all the selected modes for sensor placement is obtained by adding the contribution of each mode using the diagonal terms of $\mathbf{E_i}$. The second factor is the sensitivity of damage to measurement noise, defined in Eq. (5).

$$\left[\mathbf{S}_i^n\right] = \frac{\partial\{\Delta\alpha\}}{\partial \mathbf{X}_i} = \left([\mathbf{S}]^0\right)^+ \left(\frac{\partial\{\mathbf{e}\}^0}{\partial \mathbf{X}_i}\right) \tag{5}$$

Where $\{\Delta\alpha\}$ is an elemental stiffness change given by $\{\tilde{\alpha}\} - \{\alpha\}$, $\mathbf{X_i}$ is a noise vector corresponding to noise contributing to the selected modes, $\mathbf{S_i^n}$ is the sensitivity of change in $\{\Delta\alpha\}$ due to a unit measurement noise, and $\{\mathbf{e}\}$ is the modal data change vector containing the differences between the selected eigenfrequencies and mode shapes at the corresponding instrumented degrees of freedom, *np*, of the structure before and after the occurrence of damage, the superscript "0" represents the noise-free value. The vector $\{\mathbf{e}\}$ can be obtained using Eq. (6).

$$\{\mathbf{e}\} = [\mathbf{S}]\{\Delta\alpha\} \tag{6}$$

where the sensitivity matrix $\mathbf{S}$ is obtained by considering the contribution of the selected modes to the sensitivity matrix $\mathbf{S_i}$ defined in Eq. (2). The first partial derivatives of the eigenfrequencies and mode shapes with respect to the noise vector $\mathbf{X_i}$ were derived by Xia and Hao (2000). For the selected mode shapes, $\mathbf{S_i^n}$ is a matrix having dimensions of *np x ne*, where *ne* is the number of elements of the structural model, therefore each row of the $\mathbf{S_i^n}$ matrix represents the influence of noise from the measured data at the kth degree of freedom. The noise sensitivity is obtained by

$$\{\mathbf{S}^n\} = \sum_{i=1}^{ne}\{\mathbf{S}_i^n\} \tag{7}$$

The damage measurability for a structural model is determinated as the ratio of $\mathbf{F}$ to $\mathbf{S^n}$, where the vector $\mathbf{F}$, defined in Eq. (8), is the summation of all measured modes *nm* of $\mathbf{E_i}$ defined in Eq. (4).

$$\{\mathbf{F}\} = \sum_{i=1}^{nm}\{diag(\mathbf{E}_\mathrm{i})\} \tag{8}$$

The first factor in the definition of the damage measurability is related to the contribution of each measured degree of freedom to the FIM as a distribution of strain energy, and the second factor quantifies the influence of the measurement noise on each instrumented degree of freedom, therefore, damage measurability values with high damage sensitivity values and low measurement noise sensitivity values must be selected to optimally locate sensors.

4. DAMAGE INDEX METHOD

The selection criteria for this method are based on previous research work that can be summarized as follows. Tang and Leu (1991) showed that changes in the mode shapes of the structure were more sensitive indicators of damage than natural frequencies. Pandey, Biswas and Samman (1991) demonstrated the use of changes in the curvature of the mode shapes to detect and locate damage; they also found that both the modal assurance criterion (MAC) and the coordinate modal assurance criterion (COMAC) were not sensitive enough to detect damage in its earlier stages. Chance, Tomlinson and Worden (1994) found that measuring curvature directly using strain measurements gives very improved results than those of the curvature calculated from the displacements. Also, Chen and Swamidas (1994) found that strain mode shapes facilitated the location of a crack in a cantilever plate using finite element method simulation. Yam *et al*. (1996) have found that the strain mode shape is more sensitive to structural local changes than the displacement mode shape. Quan and Weiguo (1998) showed that, for the steel deck of a bridge, the curvature mode shapes are the best among three damage recognition indices based on mode shapes (the COMAC, the flexibility, and the curvature mode shape).

The DI method is adopted here for damage detection. This method has been extensively used in previous damage detection studies showing better performance over other existing damage detection methods (Farrar and Jauregui, 1996; Barroso and Rodriguez, 2004; Li, 2010). The DI method was developed by Stubbs and Kim (1994) to detect the existence and location of damage in a structure and is based on the assumption that strain energy stored in damaged regions will decrease after the occurrence of damage. The damage index β_j is estimated by the change of the curvature of a particular mode shape, which is related to mode strain energy changes at location j, β_j is then defined in Eq. (9). The complete derivation of the method can be found in Alvandi and Cremona (2006).

$$\beta_j = \frac{1}{n}\sum_{i=1}^{n} \frac{\int_{a_j}^{a_{j+1}} \left(\partial^2\psi_i^* / \partial x^2\right)^2 dx \int_0^L \left(\partial^2\psi_i / \partial x^2\right)^2 dx}{\int_{a_j}^{a_{j+1}} \left(\partial^2\psi_i / \partial x^2\right)^2 dx \int_0^L \left(\partial^2\psi_i^* / \partial x^2\right)^2 dx} \tag{9}$$

where $\partial^2\psi_i/\partial x^2$ and $\partial^2\psi_i^*/\partial x^2$ are the second derivatives of the *i*th mode shape before and after the occurrence of damage, respectively, n is the number of the selected mode shapes, L is the length of the beam element in which damage is being evaluated, and a_j and a_{j+1} are the limits of this beam element. The damage index for the selected mode shapes is obtained by adding the individual contribution to the damage index from each of the selected mode shapes.

The damage index procedure can be summarized as follows: (1) calculate the mode shapes amplitudes for the nodes where sensors are located; (2) estimate the amplitudes of the mode shapes for the nodes where no sensors are located by interpolating the instrumented nodes using cubic-spline functions; and, (3) take a second derivative of the interpolation function at each node. Finally, treating β_j as a realization of a normally distributed random variable β, a normalized damage index is computed as shown in Eq. (10).

$$\mathbf{Z}_j = \frac{\beta_j - \bar{\beta}}{\sigma_\beta} \tag{10}$$

where $\bar{\beta}$ and σ_β are the mean and standard deviation of the damage index, respectively. The *j*th substructure is defined as damaged when $Z_j>2$, which corresponds to a hypothesis testing with 95 % confidence level. The DI method is implemented in this paper using the graphical user interface DIAMOND developed at Los Alamos National Laboratory (Doebling, Farrar and Cornwell, 1997).

5. DESCRIPTION OF THE BRIDGE

The box-girder bridge used for this numerical study was presented by Yang and Wu (2002), and corresponds to a railway bridge. Having a bridge span of 30 m, Young's modulus of 28,25 GPa, per-unit-length mass (including the mass of the ballast layer) of 41,7 t/m, sectional area of 6,73 m^2 and moment of inertia of 7,84 m^4. The FE model of the bridge is composed of 30 Euler-Bernoulli elements. Each element is 1 m length based on previous work conducted by Riveros *et al.* (2010) using very flexible slender structures. The nodes at each end of the structure have vertical and free rotational DOFs. In figure 1, optimum sensor locations are shown in the upper part of the model, elements selected to perform damage detection are labeled in the squared elements. Only one supporting node has a free horizontal DOF as depicted in figure 1.

6. NUMERICAL SIMULATION

It is still challenging to implement a damage detection method using a few number of instrumented nodes; the FE model presented in this paper has 90 DOFs and the maximum number of sensors that will be selected from optimum sensor placement to perform damage detection is 10, on the other hand, only vertical components of the vertical mode shapes are considered for this numerical implementation. The DI method can perform damage detection using this limited information, but for other damage detection methods which require the use of stiffness matrix and mass matrix of the whole structure,the number of unmeasured DOFs makes this analysis more complicated, almost impossible, especially when the measured information is used to assemble the stiffness matrix for the damage state.

The number of modes that can be identified from the bridge should be considered when deciding the final distribution of sensors. When continuous damage monitoring is the main objective of the modal identification test, ambient vibration test, rather that forced vibration test or free vibration test is the most affordable and suitable test, especially traffic loading can be used as an ambient excitation source for railway bridges. Traffic loading on railway bridges has been studied at the University of Tokyo by Miyashita *et al.* (2005) showing that cyclic external loads of the bogies generate forced vibration and the frequency is proportional to the train velocity. Traffic loading can be modeled under some assumptions as a stationary broadband force leading to the possibility to extract the free response of the structure, which can be used to extract its dynamic features.

Higher modes can also be extracted in a structure when forced vibration testing is used, but in real applications, it is very difficult to apply sufficient artificial excitation to surpass the ambient vibration forces acting on the bridge. The advantage of using ambient vibration sources is that not only they are cheap, but also they are truly representative of the real excitation to which the bridge is subjected during its service life, but, on the other hand, the use of ambient excitation sources for continuous monitoring implies the use of limited information, as a result, only few low frequency modes can be used for damage detection. Because the main goal of this study is the numerical implementation of a continuous damage monitoring system for beam-like structures, we assume herein that traffic will be the main excitation source. This type of loading acts primarily to excite the vertical modes of the bridge. Thus, in this study we focus on a selected number of modes dominated by vertical motions in order to simulate the use of sensors capable of measuring only vertical components. Feng *et al.* (2004) developed a neural network-based system identification technique for

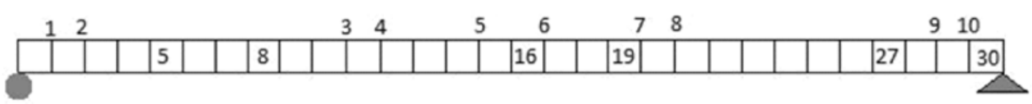

Figure 1. Optimum sensor configuration and damaged elements

model updating using traffic-induced vibrations collected from a box-girder bridge located in Irvine, California. They emphasized that the determination of input forces during an ambient vibration test are difficult to estimate. Therefore, modal parameters are usually derived from vibration measurements without requiring information about input loads. Five vertical modes using vibration data were identified by Feng *et al.* (2004).

Figure 1 depicts the resulting sensor configuration obtained from the numerical implementation of the DM method. The first 5 vertical modes are selected for this implementation based on the work presented by Feng *et al.* (2004). The DI method is then used to identify the occurrence of damage. Two levels of damage are selected in this paper. Therefore, structural damage is modeled by 10 % (case 1) and 15 % (case 2) stiffness reduction of the elements defined in table 1.

Table 1. Damage scenarios

Damage scenario	DS1	DS2	DS3	DS4	DS5	DS6
Damaged elements	5	8	16	19	27	30

Damage is considered to be successfully detected if a sensor channel shows a large variation in the strain energy before and after the occurrence of damage according to Eq. (10). The damage detection results are shown in figures 2, 3, 4 and 5. Damage is successfully identified if the normalized damage index presented in Eq. (10) is larger than 2. Figure 2 shows that the sensor location 4 identifies the occurrence of damage for scenario DS1. It can be seen that all the scenarios presented in this paper for both cases are correctly identified. Actually, when damage occurs in a simply supported bridge all mode shapes are affected and therefore the damaged region induces a reduction of the measured modal strain energy in other regions allowing the identification of the occurrence of damage. It is also important to note that negative values can be obtained during damage identification. Basically, those cases represent large differences in the curvature of the mode shapes between healthy and damage states. The structure presented in this paper corresponds to an isostatic structure; it means that the occurrence of damage significantly modifies its mode shapes. For example, the fundamental mode shape of a simply supported beam corresponds to a half-cycle of a sinusoidal function with no inflection points in the mode shape, and considering that the proposed method is based on modal strain energy, the detection of damage occurrence is more challenging. On the other hand, a continuous beam or a frame structure, defined as hyperstatic structure, contains more inflection points in its mode shapes. The fact is that damage detection becomes more challenging for isostatic structures in terms of collapse of the structure and transmission of loads to the supports and therefore this study is oriented to that case. Failure in an element of an isostatic structure may disrupt significantly the path that follows the load applied to the structure and even cause the collapse of the structure. On the other hand, failure of an element in a hyperstatic structure may modify the way in which loads are transmitted to the supports, but the stability of the structure may not be compromised. Finally, negative values can be treated by using embedding machine learning techniques.

An additional study is performed using an evenly distributed sensor configuration as shown in figure 6. The main objective of this final case (case 3) is to show the advantages of using an optimum sensor placement method. The damage detection results for the case 3 are presented in figures 7 and 8. It is possible to observe that the quality of the damage identification results is affected by the lack of an optimum sensor placement method. The number of non-identified cases is expected to increase if the number of sensors is reduced.

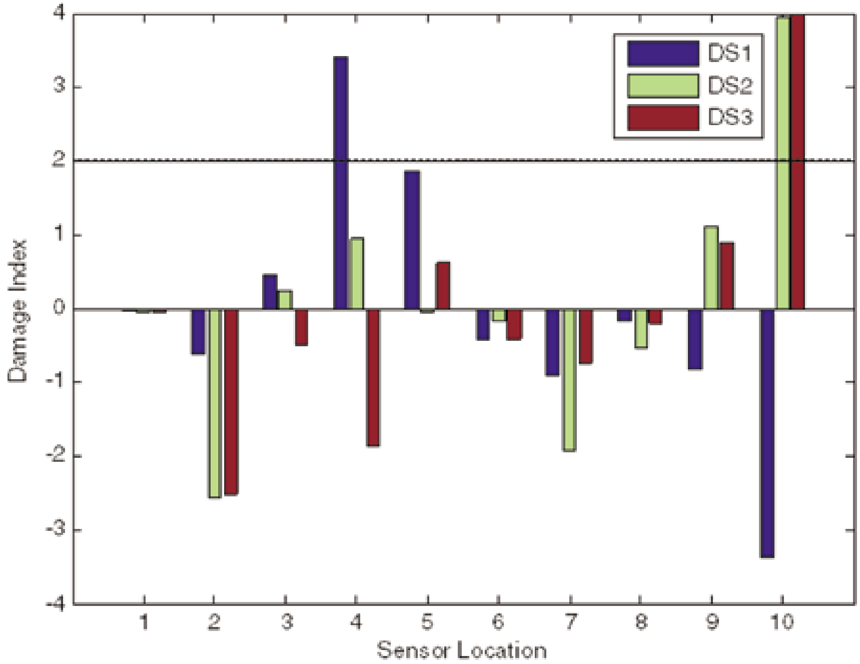

Figure 2. Damage detection results case 1 (DS1, DS2, DS3)

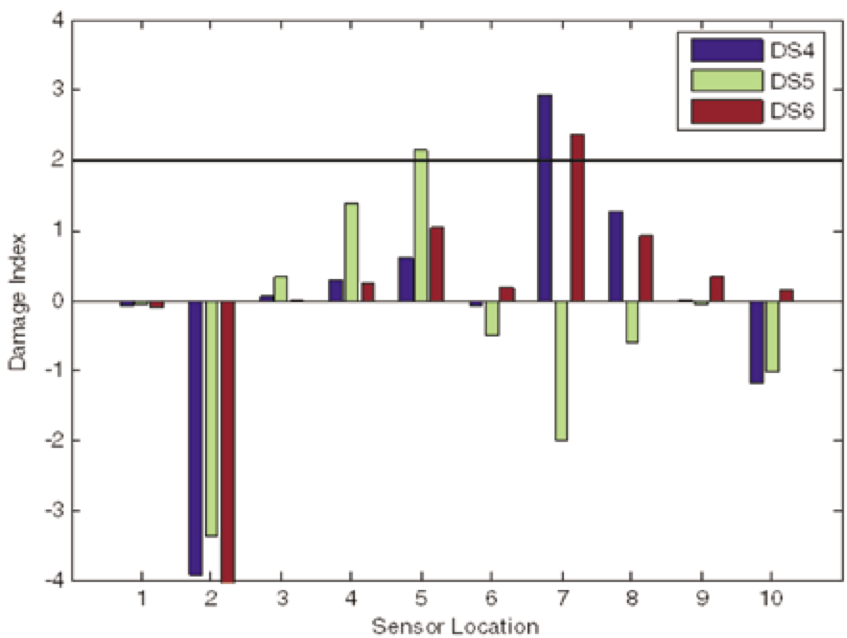

Figure 3. Damage detection results case 1 (DS4, DS5, DS6)

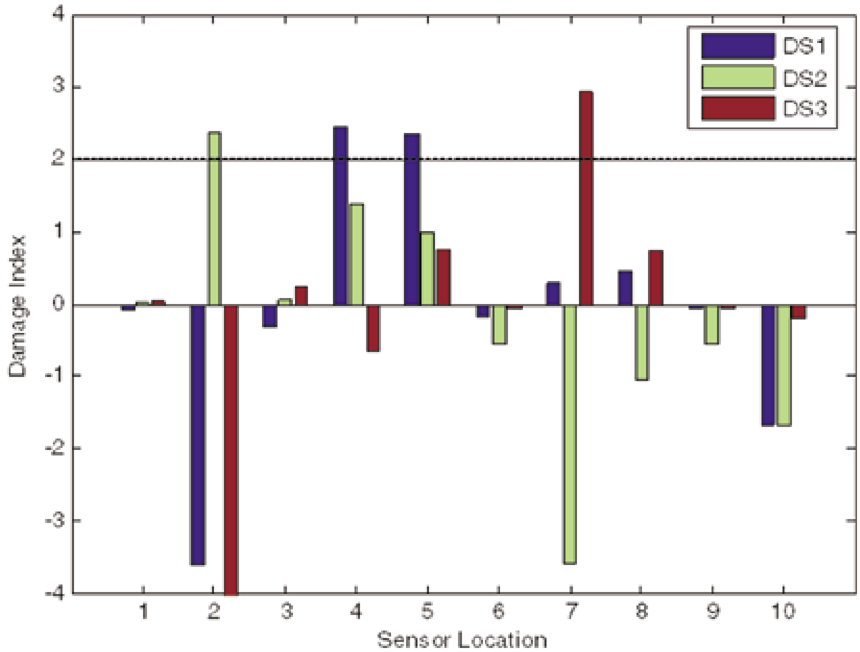

Figure 4. Damage detection results case 2 (DS1, DS2, DS3)

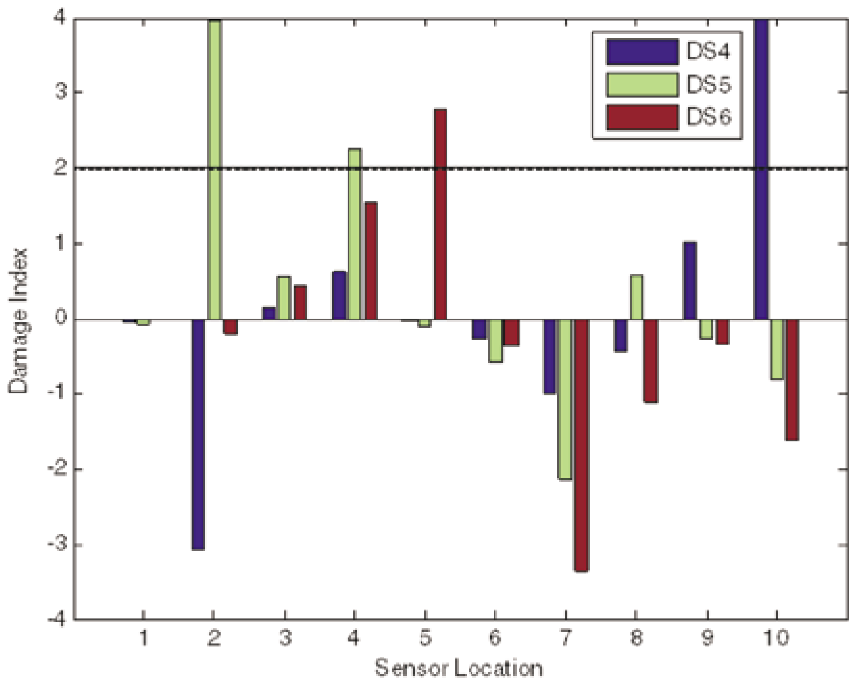

Figure 5. Damage detection results case 2 (DS4, DS5, DS6)

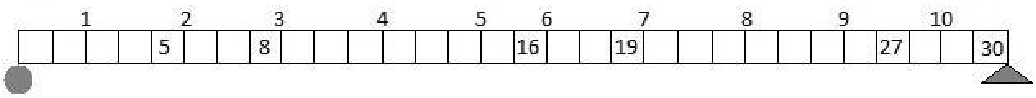

Figure 6. Evenly distributed sensor configuration

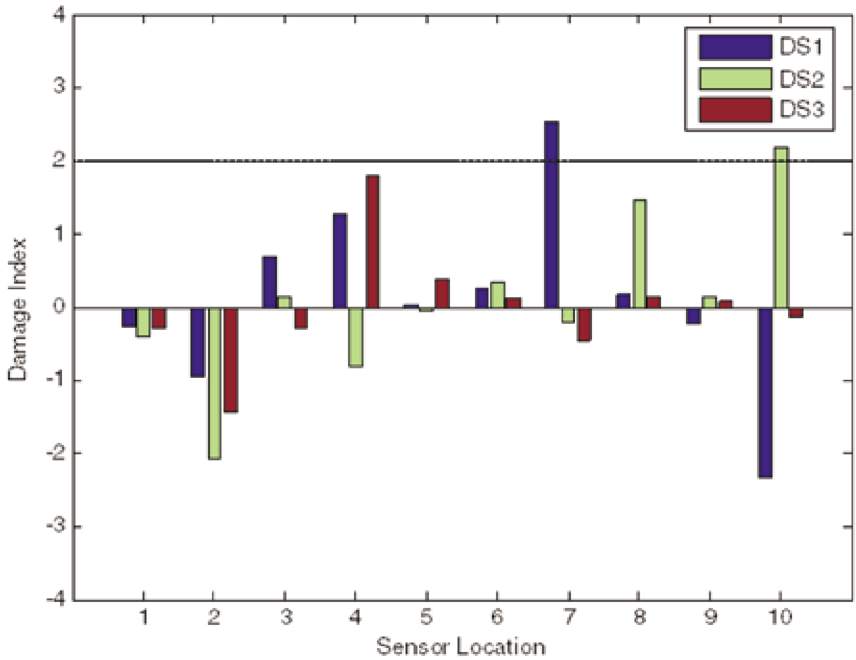

Figure 7. Damage detection results case 3 (DS1, DS2, DS3)

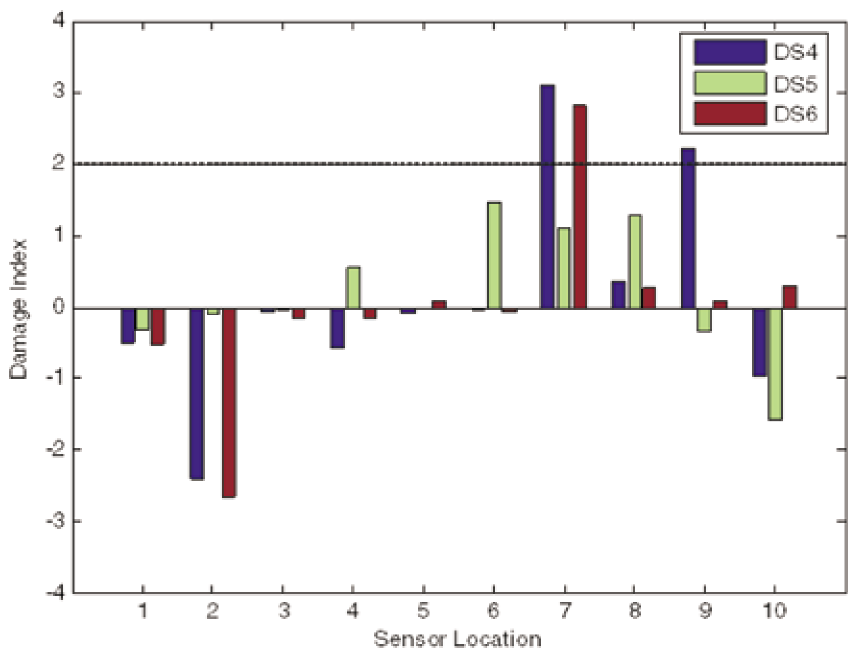

Figure 8. Damage detection results case 3 (DS4, DS5, DS6)

7. CONCLUSIONS

In this paper, a modal strain energy-based approach for fault detection in beam-like structures was presented. An optimum sensor location method developed by Xia and Hao (2000) was numerically implemented in a FE model of a box-girder bridge and a damage detection study was performed using the DI method. The basic assumption of vibration-based damage detection is that structural damage will significantly alter the stiffness, mass, or energy dissipation properties of a system, which, in turn, alter the measured dynamic response of that system. The DI method could identify the occurrence of damage for the damage scenarios presented in this paper. The results presented show the good performance of the proposed strain energy-based approach and agree well with the results obtained by Li (2010). The use of the higher modes was avoided in order to present more realistic results. In addition, damage detection was conducted by considering only vertical contributions of the selected mode shapes. It was also numerically shown that optimum sensor placement provides a significant improvement of damage detection results. The excellent anti-noise performance of modal strain energy-based methods was already demonstrated by Alvandi and Cremona (2006). Therefore, the methodology presented in this paper is a potential tool for structural health monitoring of beam-like structures, such as the group of simply supported bridges that compose the Metro system in Medellin.

Identification and damage detection techniques that are able to locate damage based on measured data collected from real structures still seem a long way from reality. A false positive (false alarming of fault) is one of the main concerns when developing a robust damage detection methodology. Therefore, for practical applications of the method presented in this paper it will be necessary to use embedding machine learning techniques in order to identify false positives. In addition, the development of low-cost and rapid-to-deploy wireless structural monitoring systems having embedded damage detection algorithms as proposed by Weng *et al*. (2008) will provide robust and affordable solutions for vibration-based damage detection implementations. Another possible scenario is that damage location using low frequency vibration is undertaken to identify those areas where more detailed local inspection should be concentrated.

NOMENCLATURE

B = determinant of the FIM as a distribution of strain energy

E = contribution to the FIM as a distribution of strain energy

F = summation of all measured modes

S = sensitivity matrix

X = noise vector

Z = normalized damage index

$\{\mathbf{e}\}$ = modal change vector

$\{\alpha\}$ = undamaged initial stiffness parameter

$\{\tilde{\alpha}\}$ = damaged initial stiffness parameter

$\{\Delta\alpha\}$ = elemental stiffness change

β = damage index

λ = undamaged eigenfrequency

$\bar{\lambda}$ = damaged eigenfrequency

ϕ = undamaged eigenvector

$\tilde{\phi}$ = damaged eigenvector

ψ = undamaged mode shape

ψ^* = damaged mode shape

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